Statistical Predictive Modeling and Compensation of Geometric Deviations of Three-Dimensional Printed Products

Geometric fidelity of 3D printed products is critical for additive manufacturing (AM) to be a direct manufacturing technology. Shape deviations of AM built products can be attributed to multiple variation sources such as substrate geometry defect, disturbance in process variables, and material phase change. Three strategies have been reported to improve geometric quality in AM: (1) control process variables $x$ based on the observed disturbance of process variables $\Delta x$, (2) control process variables $x$ based on the observed product deviation $\Delta y$, and (3) control input product geometry $y$ based on the observed product deviation $\Delta y$. This study adopts the third strategy which changes the computer-aided design (CAD) design by optimally compensating the product deviations. To accomplish the goal, a predictive model is desirable to forecast the quality of a wide class of product shapes, particularly considering the vast library of AM built products with complex geometry. Built upon our previous optimal compensation study of cylindrical products, this work aims at a novel statistical predictive modeling and compensation approach to predict and improve the quality of both cylindrical and prismatic parts. Experimental investigation and validation of polyhedrons indicate the promise of predicting and compensating a wide class of products built through 3D printing technology.

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1 Introduction

AM or 3D printing directly fabricates physical products from a 3D CAD model by layered manufacturing processes. Since AM adds material layer by layer to construct products, this technique theoretically enables the direct printing of products with extremely complex geometry. Geometric complexity does not affect building efficiency, and no extra effort is necessary for molding construction or fixture tooling design, making 3D printing one of the most promising manufacturing techniques [1–5].

Despite these promising features, dimensional accuracy control remains a major bottleneck for the application of 3D printing in direct manufacturing [6–9]. Shape deviations of AM built products can be attributed to multiple variation sources. For instance, Cohen and Lipson [10] summarized three classes of process uncertainties that could diminish the geometric fidelity of fabricated parts: situational, environmental, and build-material uncertainties with examples of substrate geometry defect, disturbance in process variables, and material phase change in each class, respectively.

As summarized in Table 1, three feedback control strategies have been reported to reduce process uncertainties and improve geometric quality in AM: (1) control process variables $x$ based on the observed disturbance of process variables $\Delta x$, (2) control process variables $x$ based on the observed product deviation $\Delta y$, and (3) control input product geometry $y$ based on the observed product deviation $\Delta y$. For instance, in the first category, Hu et al. [11] studied the real-time sensing and control of metal powder delivery in laser-based AM. To achieve a controllable powder delivery, a closed-loop control system based on infrared image sensing was built for control of the heat input and size of the molten pool. Song and Mazumder [12] monitored the melt pool temperature of a laser cladding process with a dual-color pyrometer. A state-space dynamic model was established to relate the laser power with the melt pool temperature. The closed-loop process tracked and stabilized the melt pool temperature to a reference temperature profile.

In the second category, Heralic et al. [13], to obtain a flat deposition surface in a laser metal-wire deposition process, controlled the offset of the robot in the vertical direction based on the 3D scanned data. The deviations in the layer height were compensated by controlling the wire feed rate on next deposition layer through iterative learning control. Cohen and Lipson [10] argued that monitoring the process variables $\Delta x$ could be limited to the extent to which process uncertainties can be detected and corrected. They developed geometric feedback control to directly manipulate the location of deposited matter to compensate for geometric inaccuracies based on the observed whole-part geometry.

In the third category, Tong et al. [14,15], to control detailed features along the boundary of the printed product, changed the CAD design to compensate for shrinkage, and used polynomial regression models to analyze the shrinkage in X, Y, and Z directions separately. However, prediction of deformation based on the shift of individual points is independent of the geometry of the product, which is not consistent with the physical manufacturing process.
For complete control of all local features around the perimeter of a AM built part, Huang et al. [8] established a generic approach to model and predict part deviations and subsequently derived an optimal compensation plan to achieve dimensional accuracy. The essence of this new modeling approach is to transform in-plane (x–y plane) geometric errors into a functional profile defined on the polar coordinate system (PCS). This representation decoupled the geometric shape complexity from the deviation modeling and a generic formulation of shape deviations can thus be achieved. The developed approach was demonstrated both analytically and experimentally in a stereolithography process (SLA). Experimental results demonstrate the ability of the proposed compensation approach to achieve an improvement of one order of magnitude in reduction of geometric errors for cylindrical products. However, this study did not demonstrate how the established method can be extended to noncylindrical products.

Huang et al. [9] attempted to connect the model for cylindrical shape with the model for polygon shapes in a unified modeling framework. The proposed model contains a basis function for cylindrical shape and a cookie-cutter basis function to carve out the polygon shape from the cylindrical shape. Experimental and analytical studies of square, pentagon, and dodecagon shapes were conducted to verify the unified model. However, the model fitting results, though good for square and pentagon shapes, needed improvement for polygons with large number of sizes. In addition, individual models were fitted for each shape in Ref. [9], as opposed to a single integrated model. No compensation studies were provided to validate the proposed models.

This study follows the work in Refs. [8] and [9] to establish a specific system-level approach to statistically predict the quality of a wide class of product shapes, particularly considering the vast library of AM built products with complex geometry, and to compensate the geometric deviations of AM built products. Following Sec. 1, Sec. 2 briefly reviews the previously developed modeling approach and results in Refs. [8] and [9]. In Sec. 3, polygon shape deviations are experimentally re-investigated in order to establish a consistent model beyond Ref. [9]. Section 4 presents the system-level modeling approach to predict and compensate the quality of product for a wide class of products. A compensation experiment is conducted to validate for the proposed modeling method as well. Conclusion and future work are given in Sec. 5.

### 2 Deviation Modeling of Cylindrical Products—Summary of Our Previous Work

In our previous study to describe the deviation of a 3D printed product from its intended shape [8], we chose the PCS using \((r, \theta, z)\) over the cartesian coordinate system (CCS) using coordinates \((x, y, z)\). As shown in Fig. 1 [8], the product is specified by the function \(r(\theta, r_0(\theta, z), z)\) at the boundary, and the shape deviation is represented as

\[
\Delta r(\theta, r_0(\theta, z), z) = r(\theta, r_0(\theta, z), z) - r_0(\theta, z) \quad (1)
\]

The cartesian representation has been previously studied in the literature [14,15]. It faces a practical issue of correctly identifying shape deviation. As shown in Fig. 1(b), for a given nominal point \(A(x, y, z)\), its final position \(A'\) is difficult to identify after deformation. A practical solution is to fix the \(x\) or \(y\) coordinate and study the deviation of the other coordinate (\(\Delta y\) or \(\Delta y\) in Fig. 1(b)). Choice of either direction could lead to different deformation results. Another method is to study deformation along three directions separately [14,15]. But the apparent correlation of deformation among the three directions cannot be captured, potentially leading to prediction error.

In contrast, our definition of radius deviation naturally captures geometric errors and is convenient for visualizing patterns, as shown in Fig. 2(a) the in-plane geometric errors of cylindrical products built by the SLA processes [8].

The essence of this representation is to transform in-plane (x–y plane) geometric errors into a functional profile defined on the interval \([0, 2\pi]\). This representation decoupled the geometric shape complexity from the deviation modeling, and a generic formulation of shape deviations can thus be achieved.

Since the ultimate goal of the third strategy of feedback control is to change the CAD input \(r_0(\theta, z)\) to compensate for product deviations, we modeled the functional dependence of in-plane error \(\Delta r(\theta, r_0(\theta))\) denoted as \(f(\theta, r_0(\theta))\) hereafter on \(r_0(\theta)\) as

\[
f(\theta, r_0(\theta)) = f_1(\psi) + f_2(\theta, r_0(\theta)) + \epsilon_0 \quad (2)
\]

where function \(f_1(\psi)\) represents average deviation or trend independent of location variable \(\theta\), which is often related to the volumetric change of the product. Function \(f_2(\theta, r_0(\theta))\) is the location-dependent deformation in addition to the trend. Term \(\epsilon_0\) represents high frequency components that add on to the main harmonic trend.

We interpreted \(f_1(\cdot)\) as a lower order term and \(f_2(\cdot, \cdot)\) as a higher order component of the deviation function. With the data shown in Fig. 2, it is natural to conduct Fourier series expansion of \(f_2(\cdot, \cdot)\)

\[
f(\theta, r_0(\theta)) = c + \sum_{k} \{a_k \cos(k\theta) + b_k \sin(k\theta)\} + \epsilon_0 \quad (3)
\]

and the deviation of cylinders with various sizes at location \(\theta\) is given as

\[
E(\hat{f}(\theta, r_0(\theta))|\theta) = x_0 + \beta_1(r_0 + x_0)^a + \beta_2(r_0 + x_0)^b \cos(2\theta) \quad (4)
\]

where \(x_0\) is a constant effect of over exposure for the process we study, which equivalent to a default compensation \(x_0\) applied to every angle in the original CAD model. Note that the in-plane (x–y) deformation error is of primary interest and the coordinate \(z\) is ignored.

Model (4) is fit to our data for 0.5 in. in. 2 in., and 3 in. radius cylinders, and we adopt a Bayesian approach with a weakly informative prior for \(x_0, \log(x_0) \sim N(0, 1)\) a priori. The posterior predictive distribution of the deformations generated by this model is presented in Fig. 2(a) (dashed lines).

The optimal compensation \(x'(\theta)\) to reduce deviation around the perimeter of cylindrical products in SLA processes is
This model is predictive and the validation experiment on the compensation strategy shows that the deviation of a cylinder with 2.5in radius can be dramatically reduced (Fig. 2(b)). Note that no experiment on the cylinder with 2.5in is done before compensation, which demonstrates the predictability of the developed model.

3 Experimental Study of Polygon Shape Deviations

Although model (2) is quite generic and model (3) seems easily extendable beyond cylinders, in this section we will first experimentally investigate the deviations of AM built products with (in-plane) polygon shapes. There are two reasons to choose polygon shapes: (1) Polygons have sharp corners causing more significant manufacturing challenge than cylinders; (2) When the number of polygon sides increases, the polygon model should approach to the cylinder model defined in Eq. (4); and (3) Combination of sections of cylindrical and polygon shapes will lead to a wider classes of products. Polygon experiments, therefore, provide an opportunity to test the generality and validity of the proposed approach.

3.1 AM Experiments—SLA Process. The AM process used in this work is a variant of the SLA process, called mask image projection SLA (MIP-SLA) [16]. As can be seen in Fig. 3, the MIP-SLA machine has liquid resin stored in a tank configured with a platform that can move vertically with precision. During the printing process, the surface of the resin is exposed to light, which triggers the resin solidification. Control of light exposure area and intensity is achieved through a digital micromirror device (DMD) that receives commands from the sliced cross section of stereolithography (STL) files for each layer. The platform in the tank moves down with the predefined thickness for printing the next layer when the previous layer is solidified. This building process is repeated until the last layer is built. MIP-SLA is considered
as an inexpensive fast AM technique, since it is capable of building multiple objects simultaneously.

In this experiment, we are interested to further investigate the shape deviation of polygon parts with sharp corners. We are using a commercial MIP-SLA platform, the ULTRA\textsuperscript{®} machine from EnvisionTec, to conduct all experiments. The material used in this process is SI500 resin that shrinks approximately 2\% in each phase transition. Specification of the manufacturing process is shown in Table 2.

### 3.2 Experimental Design and Observations

The experimental factors considered for polygon experiments include the number of sides and the size of a polygon. When the number of sides increases, the corner angles of a polygon vary. The polygon size is defined as the radius of its circumcircle (see examples in Fig. 4). The purpose of using a circumcircle is to connect the polygon experiments with cylinder experiments done before.

Three polygon shapes are investigated: square, regular pentagon, and dodecagon. As shown in Table 3, four squares with side lengths of 1 in., 2 in. (before and after repair), and 3 in. (with circumcircle radii of 1 in./\sqrt{2}, 2 in./\sqrt{2}, and 3 in./\sqrt{2}), two regular pentagons with circumcircle radii 1 in. and 3 in., and one regular dodecagon (polygon with 12 sides) with circumcircle radius of 3 in. are designed and fabricated in a MIP-SLA machine. The purpose of fabricating a dodecagon is to check how well a model can be generalized to different polygons and how well a polygon model can be approximated by a cylindrical shape when the number of polygon sides is large. Since we discovered in Ref. [9] that the MIP-SLA machine settings were changed after repair, new experiments on cylindrical parts are also included for experimentation in this study with \( r = 1 \) in. and 2 in.

To facilitate the identification of the orientation of test parts during or after the building process, a nonsymmetric cross with line thickness of 0.02 in. is added on the top of them (see Fig. 4(b)). All test parts have height of 0.25 in. The 3D CAD models are exported to STL format files, which are then sent to the SLA machine.

After the building process, all test parts are measured using a Micro-Vu precision machine. In order to reduce human errors, we follow the same measuring procedure for each test part. For simplicity of measurement, we choose the center of the cross to be the origin of the measurement coordinate system in the Micro-Vu machine. The boundary profile is fitted using the splines in the metrology software associated with the Micro-Vu machine. The obtained measurement data are then converted to polar coordinates for deviation modeling and analysis. It must be noted that the part orientation is kept the same during the building and measurement processes. Figures 5–7 show the measured deviation profiles (solid lines) for three kinds of polygons and cylinders presented in the PCS.

By comparing the deviation profiles of cylindrical and square shape before and after the machine repair (Fig. 2 versus Fig. 5), it is clear that (1) the repaired MIP-SLA machine tends to overcompensate the product shrinkage and lead to positive shape deviation; and (2) The systematic deviation patterns in square shape clearly differ before and after the machine repair. Therefore, in Sec. 4, we intended to identify basic deviation patterns consistent across different shapes and provide an integrated model for prediction and compensation.

### 4 Statistical Predictive Modeling of Geometric Deviation of AM Built Parts

Following the same notation in model (2), we denote \( f(\theta, r_0(\theta)) \) as the in-plane shape deviation of 3D printed product presented in the PCS. The Fourier expansion of \( f(\theta, r_0(\theta)) \), though works well for the cylindrical shape, clearly encounters the
difficulty of model fitting for polygon shapes, i.e., too many high-order Fourier basis terms have to be included in order to capture the sharp transition at the polygon vertices. There are two disadvantages: (1) model overfitting which causes poor predictability; and (2) lack of physical insights gained from the model.

Xu et al. [18] studied the experimental data of Figs. 2 and 5 and proposed the hypothesis of shape deviation due to different shrinkage factors along different directions. Xu et al. [18] explained the mechanism considering (i) over or under exposure, (ii) light blurring, and (iii) phase change induced shrinkage or expansion. A shape deviation model based on shrinkage factors was established to quantify the effects of three sources for both cylindrical and square shapes with various sizes. The attempt assists to understand the physical insights of the AM processes. Since our objective is to control input product geometry $y$ based on the observed product deviation $\Delta y$, we aim at identifying the statistical patterns in the deviation profiles, rather than the specific physical mechanisms contributing to the deviation profiles.

### 4.1 Statistical Modeling Strategy

Our main strategy of connecting the cylindrical shape model to polygon models is to treat a polygon as being cut from its circumcircle as shown in Fig. 4. This modeling strategy implies that a generic deviation profile model for cylinders or polygons contains at least two major basis functions: (1) basis model $g_1$ for cylindrical shape deviation profile and (ii) basis model $g_2$ for a cookie-cutter function. This concept extends our previous model in Eq. (2) into

$$f(\theta, r_0(\theta)) = g_1(\theta, r_0(\theta)) + g_2(\theta, r_0(\theta)) + g_3(\theta, r_0(\theta)) + \epsilon \quad (6)$$

where $g_3$ term denotes the remaining feature not captured by $g_1$ and $g_2$, if there is any.

Following the updated model (6), we will identify $g_1$ and $g_2$, respectively.

#### Table 3 Polygon experimental design

<table>
<thead>
<tr>
<th>Cross section geometry</th>
<th>Circumcircle radius</th>
<th>Process condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>$r = 1 \text{ in.}/\sqrt{2}$; $2 \text{ in.}/\sqrt{2}$; $3 \text{ in.}/\sqrt{2}$</td>
<td>Before machine repair</td>
</tr>
<tr>
<td>Square</td>
<td>$r = 2 \text{ in.}/\sqrt{2}$</td>
<td>After machine repair</td>
</tr>
<tr>
<td>Regular pentagon</td>
<td>$r = 1 \text{ in.}; 3 \text{ in.}$</td>
<td>After repair, settings changed</td>
</tr>
<tr>
<td>Regular dodecagon</td>
<td>$r = 3 \text{ in.}$</td>
<td>After repair, settings changed</td>
</tr>
<tr>
<td>Circle</td>
<td>$r = 1 \text{ in.; 2 in.}$</td>
<td>After repair, settings changed</td>
</tr>
</tbody>
</table>

Fig. 5 Deviation profiles (solid lines) of cylinders with $r_0 = 1 \text{ in.}; 2 \text{ in.}$ and square shapes with side length $= 1 \text{ in.}; 2 \text{ in.}; 3 \text{ in.}$ (a) Cylinders and square after repair and (b) square shapes before repair.

Fig. 6 Observed deviation profiles for pentagons. (a) Two regular pentagon deviation profiles with circumcircle radii $= 1 \text{ in.}; 3 \text{ in.}$ (solid lines) and (b) printed regular pentagon with circumcircle radius $= 3 \text{ in.}$
4.1.1 Cylindrical Basis Model. Huang et al. [8] conducted thorough investigation of the cylindrical basis model 

\[ g_1(h, r_0(\theta)) \]

where \( g_1 \) was represented in Eq. (4) or

\[ g_1(h, r_0(\theta)) = x_0 + b_0(r_0 + x_0)^a + b_1(r_0 + x_0)^b \cos(2\theta) \]  

When extending this basis model to generic shape, there is a danger of over-paramaterization, that is, the total number of unknown parameters in \( g_1 \) and \( g_2 \) will be more than that can be estimated from the data. Therefore, we adopt an alternative model studies in Ref. [8]

A Bayesian procedure was implemented to draw inferences on all parameters \( \beta_0, \beta_1, \alpha, b, \) and \( \sigma \). We calculated the posterior distribution of the parameters by Markov Chain Monte Carlo (MCMC) and summarized the marginal posteriors by taking the mean, median, standard deviation, and 2.5% and 97.5% quantiles of the posterior draws in Table 4.

Due to apparent pattern change presented in Fig. 5(a), the cylindrical basis model \( g_1(h, r_0(\theta)) \) after machine repair is expected to have different coefficients \( \beta_0 \) and \( \beta_1 \). On the other hand, we tend to keep \( \alpha \) and \( b \) as the same values in Table 4 for two reasons (i) \( r_0^a \) and \( r_0^b \) in Eq. (7) represent the volumetric shrinkage varying with size \( r_0 \). For the same material, the change after MIP-SLA machine repair might be small. (2) Concern of overparamaterization limits the number of unknown parameters in the model. Therefore, the cylindrical basis function used is

\[ g_1(h, r_0(\theta)) = \beta_0 r_0^a + \beta_1 r_0^b \cos(2\theta) \]  

**Table 4 Summary of posterior draws [8]**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>2.5%</th>
<th>Median</th>
<th>97.5%</th>
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<tr>
<td>( \beta_0 )</td>
<td>-0.0047</td>
<td>4.063 x 10^{-5}</td>
<td>-0.0048</td>
<td>-0.0047</td>
<td>-0.0047</td>
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<tr>
<td>( \beta_1 )</td>
<td>0.0059</td>
<td>6.847 x 10^{-5}</td>
<td>0.0059</td>
<td>0.0059</td>
<td>0.0060</td>
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<tr>
<td>( a )</td>
<td>1.56</td>
<td>0.0084</td>
<td>1.5498</td>
<td>1.566</td>
<td>1.5819</td>
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<tr>
<td>( b )</td>
<td>1.099</td>
<td>0.0120</td>
<td>1.0755</td>
<td>1.099</td>
<td>1.1232</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>0.0019</td>
<td>2.503 x 10^{-5}</td>
<td>0.00185</td>
<td>0.0019</td>
<td>0.00195</td>
</tr>
</tbody>
</table>

![Fig. 7 Observed deviation profile for a dodecagon. (a) Regular dodecagon deviation profile (solid line) with circumcircle radius = 3 in. and (b) printed dodecagon with circumcircle radius = 3 in.](image)

**Fig. 8 Square wave functions with square, pentagon, and dodecagon deviation profiles**
where we use MOD function in the usual sense to obtain remainders (as in x MOD y = remainder of (x/y)).

Figure 9 illustrates examples of sawtooth functions. Comparing the definition of square wave function, it is clear that sawtooth wave function will also capture the sharp transitions around the vertices of polygons in the deviation profiles. It preserves the limiting property when n approaches to infinity.

To control the direction of each sawtooth (flipping upside down), we introduce an indicator function

\[ I(\theta - \phi_0) = \{\text{sign}[\sin(n(\theta - \phi_0)/2)] + 1\}/2 \]  

(10)

The alternative cookie-cutter basis using sawtooth wave function is therefore given as

\[ g_2(\theta, r_0(\theta)) = \beta_2 r_0^2 I(\theta - \phi_0) \text{saw.tooth}(\theta - \phi_0) \]

\[ = \beta_2 r_0^2 \{\text{sign}[\sin(n(\theta - \phi_0)/2)] + 1\}/2[(\theta - \phi_0)\text{MOD}(2\pi/n)] \]  

(11)

### 4.1.3 Residual Pattern Along Polygon Sides

If cylindrical basis \( g_1(\theta, r_0(\theta)) \) and cookie-cutter basis \( g_2(\theta, r_0(\theta)) \) cannot fully

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>P-value</th>
<th>Residual ( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 in. square</td>
<td>( \beta_0 )</td>
<td>( 1.5149 \times 10^{-3} )</td>
<td>( 1.313 \times 10^{-4} )</td>
<td>( 0.003297 )</td>
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<tr>
<td>1 in. square</td>
<td>( \beta_1 )</td>
<td>( 5.706 \times 10^{-4} )</td>
<td>( 3.571 \times 10^{-4} )</td>
<td>0.11</td>
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<tr>
<td>1 in. square</td>
<td>( \beta_2 )</td>
<td>( 2.343 \times 10^{-4} )</td>
<td>( 2.173 \times 10^{-4} )</td>
<td>( 0.00127 )</td>
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<tr>
<td>1 in. square</td>
<td>( \beta_3 )</td>
<td>( 1.561 \times 10^{-4} )</td>
<td>( 1.733 \times 10^{-4} )</td>
<td>( 0.000256 )</td>
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<tr>
<td>2 in. square</td>
<td>( \beta_0 )</td>
<td>( -5.470 \times 10^{-3} )</td>
<td>( 8.924 \times 10^{-5} )</td>
<td>( 0.000217 )</td>
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<tr>
<td>2 in. square</td>
<td>( \beta_1 )</td>
<td>( -8.053 \times 10^{-3} )</td>
<td>( 2.618 \times 10^{-4} )</td>
<td>( 0.00127 )</td>
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<td>2 in. square</td>
<td>( \beta_2 )</td>
<td>( 7.856 \times 10^{-3} )</td>
<td>( 1.678 \times 10^{-4} )</td>
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<td>2 in. square</td>
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<td>( 2.071 \times 10^{-3} )</td>
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<td>3 in. square</td>
<td>( \beta_0 )</td>
<td>( -9.763 \times 10^{-3} )</td>
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<td>( 0.000297 )</td>
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<td>3 in. square</td>
<td>( \beta_1 )</td>
<td>( -2.776 \times 10^{-3} )</td>
<td>( 2.640 \times 10^{-4} )</td>
<td>( 0.00127 )</td>
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<td>3 in. square</td>
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<td>( 1.266 \times 10^{-2} )</td>
<td>( 1.714 \times 10^{-4} )</td>
<td>( 0.000256 )</td>
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<tr>
<td>3 in. square</td>
<td>( \beta_3 )</td>
<td>( 2.797 \times 10^{-3} )</td>
<td>( 1.204 \times 10^{-4} )</td>
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<tr>
<td>1 in. pentagon</td>
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<td>( 0.001812 )</td>
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<tr>
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<td>( 8.255 \times 10^{-4} )</td>
<td>( 3.173 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
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<tr>
<td>1 in. pentagon</td>
<td>( \beta_2 )</td>
<td>( 8.739 \times 10^{-4} )</td>
<td>( 2.314 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
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<td>( 3.245 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
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<tr>
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<td>( 0.002768 )</td>
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<td>( 3.158 \times 10^{-5} )</td>
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<td>( \beta_2 )</td>
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<td>( 2.251 \times 10^{-5} )</td>
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<td>( \beta_3 )</td>
<td>( 2.012 \times 10^{-4} )</td>
<td>( 3.003 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
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<td>3 in. dodecagon</td>
<td>( \beta_0 )</td>
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<td>( 1.496537 \times 10^{-5} )</td>
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</tr>
<tr>
<td>3 in. dodecagon</td>
<td>( \beta_1 )</td>
<td>( -1.417587 \times 10^{-4} )</td>
<td>( 2.089859 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
<td></td>
</tr>
<tr>
<td>3 in. dodecagon</td>
<td>( \beta_2 )</td>
<td>( 7.680893 \times 10^{-5} )</td>
<td>( 1.494737 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
<td></td>
</tr>
<tr>
<td>3 in. dodecagon</td>
<td>( \beta_3 )</td>
<td>( 3.338678 \times 10^{-4} )</td>
<td>( 2.033879 \times 10^{-5} )</td>
<td>( 0.00127 )</td>
<td></td>
</tr>
</tbody>
</table>
capture the patterns in the deviation profiles, we could introduce additional $g_3(h, r_0(h))$ term if necessary. Then individual sides of a polygon may have a higher-order deviation pattern after trimming by the cookie-cutter, we could have another term $\cos[n(\theta - \phi_0)]$ or $\sin[n(\theta - \phi_0)]$ which also has the consistent limiting property.

4.2 Statistical Model Estimation

4.2.1 Initial Model Fitting for Individual Polygon Shapes. To test the proposed modeling strategy and cookie-cutter function, we first fit a statistical model for individual product shapes observed in the experiment. Given a cookie-cutter function, e.g., square wave function, the model (6) can be simplified by merging variables as

$$f(\theta, r_0(\theta)) = \beta_0 + \beta_1 \cos(2\theta) + \beta_2 \text{sign} \left( \cos(\frac{n(\theta - \phi_0)}{2}) \right) + \beta_3 \cos(n\theta) + \epsilon$$

We simply apply the least square estimation (LSE) to estimate the individual models for three types of polygons. The estimated model parameters of Eq. (12) for fabricated product are shown in Table 5. The fitted models are shown as dashed lines in Figs. 5–7, respectively. As can be seen in Fig. 5, with only three base functions, the model (12) captures the deviation pattern of square shapes very well (before machine repair), comparing with many model terms for Fourier series.

However, model (12) seems unable to explain pentagon and dodecagon well (Figs. 6 and 7). The potential reason can be attributed to process condition change. As mentioned in Sec. 3.2, pentagon and dodecagon products were fabricated after the repair of the MIP-SLA machine. The sawtooth pattern is more obvious after repair, which may cause the poor fitting for pentagon and dodecagon shapes based on model (12).

<table>
<thead>
<tr>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>Residual $\sigma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.00297$</td>
<td>$5.30065 \times 10^{-5}$</td>
<td>$1.2315 \times 10^{-9}$</td>
<td>1.36459</td>
</tr>
</tbody>
</table>

Table 6 Integrated model estimation for polygons

Fig. 10 Integrated model prediction and validation. (a) Polygon deviation profiles and model predictions and (b) Model validation by predicting dodecagon deviation profile.
4.2.2 An Integrated Model Fitting for Polygons. The initial model fitting for individual polygon shapes suggests the use of saw-tooth wave function as cookie-cutter basis model after the machine repair. In this section, we attempt to fit one integrated model (6) for all polygons fabricated after machine repair. We will use square shape with side length 2 in. and pentagons with circumcircle radius 1 in. and 2 in. to fit one model and validate the model using 3 in. polygon shape with side length 2 in. and pentagons with circumcircle radii 1 in. and 2 in. to fit one model and validate the model using 3 in. polygon shape after machine repair. We will use square tooth wave function as cookie-cutter basis model after the machine repair.

The optimal amount of compensation \( x^*(\theta) \) is defined as

\[
x^*(\theta) = -\frac{\beta_0 r_0^{1.566} + \beta_1 r_0^{1.099} \cos(2\theta) + \beta_2 r_0^2 I(\theta - \phi_0) \text{saw.tooth}(\theta - \phi_0)}{1 + 1.566 \beta_0 r_0^{1.566} + 1.099 \beta_1 r_0^{1.099} \cos(2\theta) + (x-1) \beta_2 r_0^2 I(\theta - \phi_0) \text{saw.tooth}(\theta - \phi_0)}
\]

(16)

The optimal amount of compensation \( x^*(\theta) \) for 3 in. dodecagon is illustrated in Fig. 11(a). We conducted compensation experiment by applying compensation \( x^*(\theta) \) to a 3 in. dodecagon. Figure 11(b) compares the deviation profiles before and after the compensation. The validation and compensation experiments indicate that

- The average profile deviations are reduced from 0.01389 in. to 0.00357 in., an average of 75% reduction is achieved. The modeling strategy given in Eq. (6), statistical predictive model (13), optimal compensation model (16) are generally robust and predictive.
- The training data for establishing polygon model (13) and optimal compensation model (16) only include square and pentagon shapes. The validation experiment conducted for dodecagon is therefore not within the experimental range of polygon sides. This provides greater confidence in the model robustness.
- The deviation profile after compensation still shows systematic patterns in Fig. 11(b). The possible aspects for model refinement could include: (1) improving cylindrical basis model \( g_1(\theta, r_0(\theta)) \) in Eq. (8) by collecting more cylinder data after machine repair and (2) introducing \( g_1(\theta, r_0(\theta)) \) to improve the prediction along polygons sides.

5 Conclusion

This study greatly extends our previous work on the modeling and optimal compensation of 3D printed products by developing a unified modeling and compensation method for cylindrical shape and polygon shapes. The novel statistical predictive model consists of cylindrical basis, cookie-cutter basis, and residual pattern along polygon sides. Our experimental validation and analysis show that this unified model, satisfying the limiting property of polygon shape, can predict the cylindrical shape and polygon shapes well. With training data of square and pentagon shapes, the
statistical model and optimal compensation model successfully predicted and compensated roughly 75% of the deviation of a dodecagon shape. The robustness and predictability of the modeling and compensation strategy can be further enhanced by improved model fitting and more experimental data. The developed statistical modeling and compensation methods hold the promise to extend the study to wider classes of AM built products.

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References