Conditional independence models for Gaussian random variables are algebraic varieties in the cone of positive definite matrices. We explore the geometry of these varieties in the case of Bayesian networks, with a view towards generalizing the recursive factorization theorem. When some of the random variables are hidden, non-independence constraints are needed to describe the Bayesian networks. These non-independence constraints have potential inferential uses for studying collections of random variables. In the case that the underlying network is a tree, we give a complete description of the defining constraints of the model and show a surprising connection to the Grassmannian. Algebraic prerequisites will be kept to a minimum.