1 Statistics 270: Lecture 1

1.1 Motivation:

1.1.1 What is credit risk?

The risk that an obligator does not honor his payment.

1. Arrival Risk: The uncertainty whether a default will occur or not
3. Recovery Risk: What will be recovered in case of default.
4. Market Risk: Price of bond fluctuates according to market fluctuation
5. Model Risk: associated with the model we use
6. Risk Risk: Also, whatever risk is remaining that we haven’t been able to discern but adds noise to the model.

Rare events are difficult to model from a Statistician’s point of view, not enough data to accurately estimate all of these risks.

1.1.2 What is a Credit derivative?

A financial instrument with a payoff entirely derived from the performance of an underlying asset.

A derivative puts a function payment layer on top of this instrument.

Story of two banks. Bank A lends loans to Groceries. There are certain risks from lending only to groceries that leads the Bank A to want to control their Risk. Bank B invests in some “negatively correlated” industry, like Restaurants. Bank A and B are both specialists in their investments they don’t want to start investing in the other field. Instead they choose to swap the portions of their assets so that they may control the risk. Bank A is hedged against the risk of grocery failure, by selling some of its risk and buying some share of the Restaurant market from Bank B.

Bank A  Bank B
(groceries) (Restaurants)

Credit Derivatives exist to make these work in the Market. Derivatives and Securitization allow banks to decrease their exposure.

1.1.3 Some types of Credit derivatives

1. Asset Swaps (take the returns from another asset, Libor Credit Swaps?)
2. Total Return Swaps
3. Credit Default Swaps
4. Collateralized Debt Obligations (CDOs)
Bank A will sell corporate bonds with monthly payout premiums. Consider an insurer, gets paid $c$, $c$, $c$, $c$ and a random fire occurs at time $\tau$ at which point the insurer must pay $100 \cdot \delta$. The question becomes, what will be the right amount that the Bank should charge for this insurance swap?

1.1.4 How big is this market?
The market for credit derivatives is seeing rapid growth. Size is 2 trillion dollars, expected to 4 trillion by end of 2004, it has still grown hugely.

1.2 Topics:

1. Equity Options
   - Binomial Model (discrete time intervals, studying probability)
   - Black-Scholes-Merton Model
     
     Link connecting these is Brownian Motion, Itô's formula. Feynman-Kac formula.
     
     Filtration.
     
     Risk-Neutral Valuation $\leftrightarrow$ according to a no-arbitrage model.
     
     Merton options can be priced riskless given continuous financing

2. Pricing Corporate Liabilities as Contingent Claims (structural models)
   - Intensity-Based Models (reduced form Model)
   - Recently a link of a structure Model has been shown, using incomplete information comparing corporate liabilities and Cox processes. Landau’s textbook gives examples of pricing these liabilities.

3. Credit Derivatives
   - Modeling Counterparty Risk
   - Correlated Defaults
   - Are the defaults and risks positively correlated?
   - Can you model 3rd party risk?

Famous names of Investment Banks:

\[
\begin{array}{cc}
\textit{Goldman Sachs} & \textit{Morgan Stanley} \\
A & A \\
B & B \\
C & D \\
\vdots & \vdots \\
\end{array}
\]

Given that they share investments in $A$ and $B$ then incase of default Goldman Sachs and Morgan Stanley share some risk. No one gives out data about the risk these banks hold on to, the data is difficult to obtain.
1.3 Pricing Equity Options

- Statistical Issues remain (bad models)
  They depend on rare events, so one might not have sufficient data
  Survival Data analysis, GLMs might be good models for default probability and times.
- Computational Issues. Closed form solutions do not always exist, implementing pricing
  requires computational methods
  One needs to compute these results very quickly, implemented in real time. Should be
  a way to get parameters through the market.

That should give you a good idea about the topics to be evaluated in the course.

1.3 Pricing Equity Options

An example: motivating the origin of an option. Suppose you work for IBM, have been given a
piece of documents listing benefits like 401k, etc. Say you got an option to buy IBM stock at a
discount, as an employee you can donate 10% of your earnings to buy the stock. However, you
must hold onto those shares for an entire 3 years. Being stuck with stock that one cannot sell is
quite a risk. Younjeung’s husband purchased a lot of IBM stock options, they are stuck with plenty
of worthless IBM that dropped.

IBM stock was selling at $50. Taking a Long position on the stock. If the price goes up, profit
goes up, price goes down, profit goes down. One can lose a total of $−50 if the stock drops.

But what if you purchase a put option!! this gives you a payout max(0, X − St) where X is the
strike price and St is the stock price. A right to sell the stock at X = $40 in the case where St
actually is $30 or something worse or less.

However, if St = 60 don’t utilize the put, sell your stock to the market for the complete profit.
This creates a payoff

1.3.1 Pictoral Equity Purchase

Ignoring the discount, say I buy a stock at $50 today and we ask what it will return T = 1 years
from now. Note the stock price can’t go below zero.

What happens if we buy a put for a fixed price, allowing us to collect some loss if the stock
goes below As we see, we can control some risk exposure with this derivative.

1.3.2 Types of Options:
- Long Call
  Buy at max(S_t − X, 0) return
1.3 Pricing Equity Options

- Long Put
  Sell at \( \max(0, X - S_t) \)

- Short Call: The seller of the call
  Payoff \(-\max(S_t - X, 0)\)

- Short Put: The Seller of a put
  Payoff \(-\max(0, X - S_t)\)

Who are participants in this?

1. Hedgers: Trying to control risk

2. Speculators: Trying to make bets based upon what they know
   Price of all the underlying equities might be too much to take the one bet you’d like to.

3. Arbitrageurs
   If they see prices of options do not add up, they will go for instantaneous profits by purchasing the right amount of equity, puts and calls to stabilize prices.