The missing data problems are everywhere in statistics. The EM algorithm offers a concise algorithm with sound theoretical properties in dealing with missing data under the existence of likelihood. The EM algorithm guarantees the monotonicity of the observed likelihood, which leads to at least the local optimality. However, when we have missing data in the case there is no likelihood, the estimation is getting much more complicated. Some ad-hoc approaches are given in a case-by-case basis but no unified approach with nice theoretical properties established up to now. We target at filling the gap in the estimation with missing data by providing the so-called “self-consistent” estimator which not only provides an easy-to-follow algorithm, but also boasts nice convergence and asymptotic properties.

The concept of the “self-consistent” estimator was first proposed in Efron (1967) and gained considerable interest in literature on nonparametric estimation with missing data, especially for the survival or distribution functions. We will compare the “self-consistent estimator” with EM algorithm and the ES algorithm (Elashoff and Ryan 2004), which is the algorithm for missing data with estimation equations. We will also present two unified approaches in establishing the algorithm convergence and theoretical properties of the “self-consistent” estimator. The first approach is based on contraction mapping theories with wavelet as the example; the second approach is less stringent and based on the fixed-point theories.