For high-dimensional data, particularly when the number of predictors greatly exceeds the sample size, selection of relevant predictors for regression is a challenging problem. Bayesian variable selection methods place prior distributions on the parameters along with a prior over model space, or equivalently, a mixture prior on the parameters having mass at zero. Since exhaustive enumeration is not feasible, posterior model probabilities are often obtained via long MCMC runs. The chosen model can depend heavily on various choices for priors and also posterior thresholds. Alternatively, we propose a conjugate prior only on the full model parameters and use sparse solutions within posterior credible regions to perform selection. These posterior credible regions often have closed-form representations and can be computed via existing algorithms. The approach is shown to be consistent for the true model and outperform common methods in the high-dimensional setting. We illustrate this method with applications to confounder selection in environmental epidemiology and a spatial analysis of climate data.